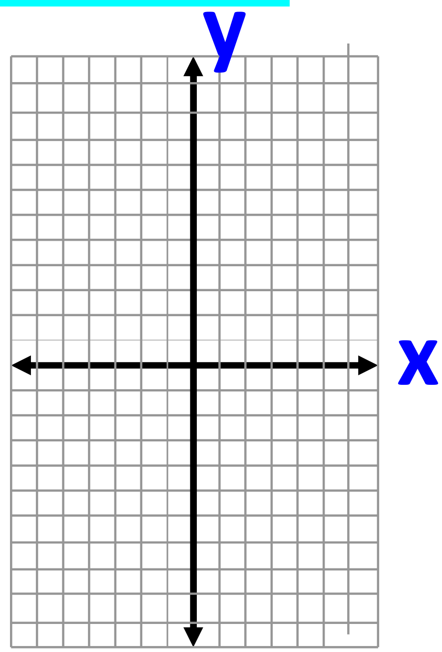
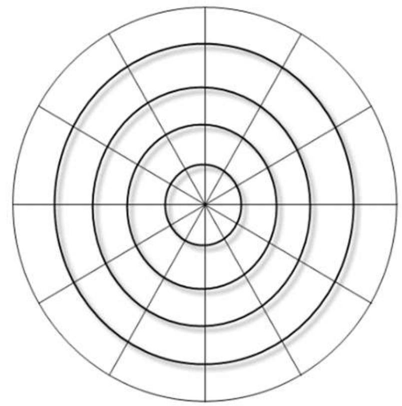
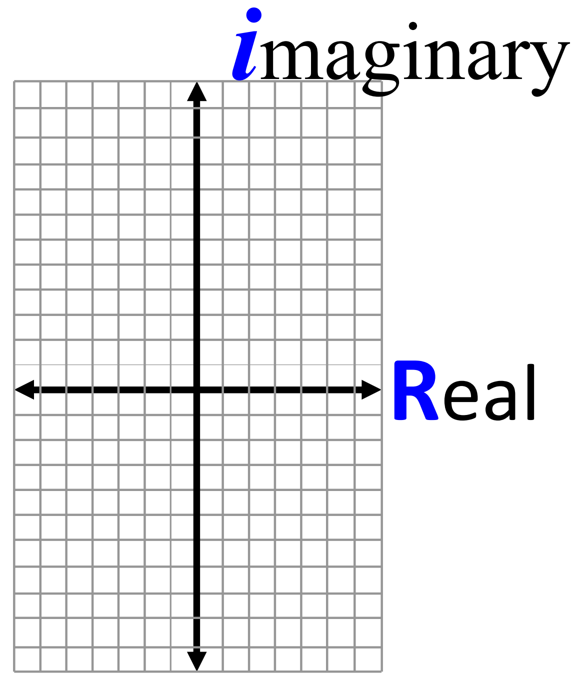
Notes: 8.3 Polar form of Complex Numbers



Rectangular Plane (x, y)



Polar Plane (r, θ)



Complex Plane

(R, *i*)

$a+bi$ → <u>rectangular form</u> of a complex number. Graphed as (a,b) in complex plane

Similar to (x, y)

Given: polar coordinates (r, θ)

r → often called the <u>absolute value</u> or the <u>**modulus**</u> *(distance)*

θ → often called the <u>amplitude</u> or <u>**argument**</u> *(angle measure)*

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a}$$

Notes: 8.3

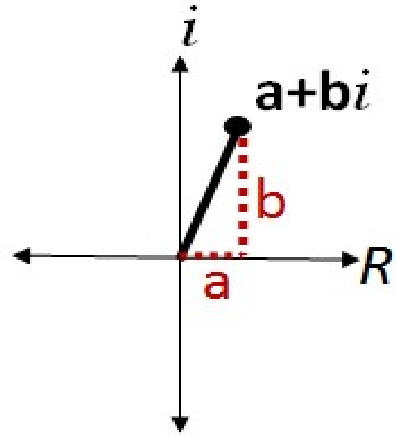
Absolute Value (modulus):

The distance between a number and zero

Absolute Value of a Complex Number

$$|a+bi| = \sqrt{a^2+b^2}$$

(r = modulus)



Polar Form of a Complex Number

$$r(\cos\theta + i\sin\theta)$$

$$r = \sqrt{a^2+b^2}$$

$$\tan\theta = \frac{b}{a}$$

Notes: 8.3

COMPARE:

coordinates

rectangular to polar

$(x, y) \rightarrow (r, \theta)$

$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

complex numbers

rectangular form to polar form

$a + bi \rightarrow r(\cos\theta + i\sin\theta)$

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a}$$

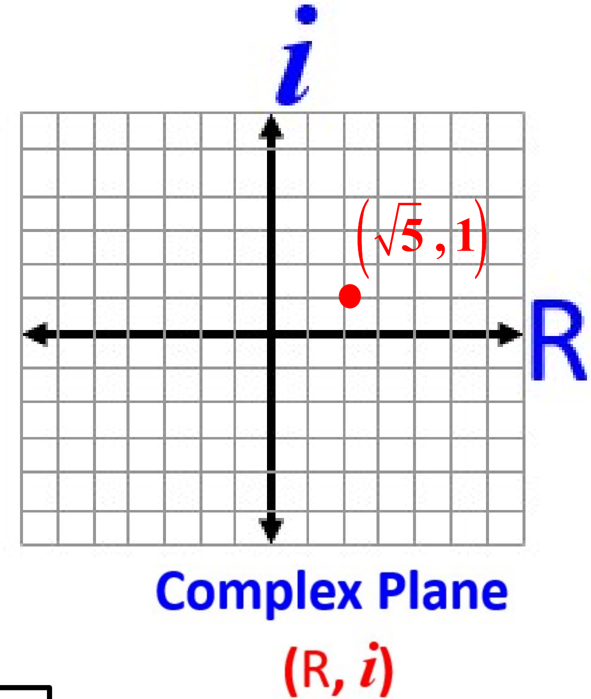
Choose tangent value from the quadrant where (x, y) and (a, b) are located

EXAMPLE 1: Graph the complex number, then find the modulus r.

$$\sqrt{5} + i$$

Graph a point at $\left(\sqrt{5}, 1\right)$

Since $\sqrt{5}$ is slightly larger than 2, move over about 2.2 and up 1.



$$\sqrt{5} + 1i \rightarrow \left(\sqrt{5}, 1\right)$$

$$a + bi \qquad a, b$$

To find the modulus, calculate r using the coefficients a and b from the given complex number (which is similar to how we used x and y previously.)

$$r = \sqrt{\left(\sqrt{5}\right)^2 + 1^2}$$

$$r = \sqrt{6} \quad \text{modulus}$$

r is the distance from the origin to the given point

$$r = \sqrt{a^2 + b^2} \quad \text{or} \quad r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{b}{a} \quad \text{or} \quad \tan\theta = \frac{y}{x}$$

EXAMPLE:

$$r\left(\cos\theta + i\sin\theta\right)$$

Write the complex number in polar form.

$$-2-2\sqrt{3}\,i$$

a **b**

Calculate r and θ using the coefficients a and b

$$r=\sqrt{a^2+b^2}$$

$$\tan\theta=\frac{b}{a}$$

$$r=\sqrt{(-2)^2+\left(-2\sqrt{3}\right)^2}$$

$$r=\sqrt{4+12}$$

$$r=\sqrt{16}$$

$$r=4$$

$$\tan\theta=\frac{-2\sqrt{3}}{-2}$$

$$\tan\theta=\sqrt{3}$$

$$\theta=\frac{4\pi}{3}$$

Choose tanθ from Quad III since the given point is located in Quad III

Therefore, $-2-2\sqrt{3}\,i \quad = \quad 4\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

EXAMPLE continued:

$$given: -2-2\sqrt{3}\,i$$

$$r=\sqrt{(-2)^2+\left(-2\sqrt{3}\right)^2}$$

$$r=\sqrt{4+12}$$

$$r=\sqrt{16}$$

$$r=4$$

$$\tan\theta=\frac{-2\sqrt{3}}{-2}$$

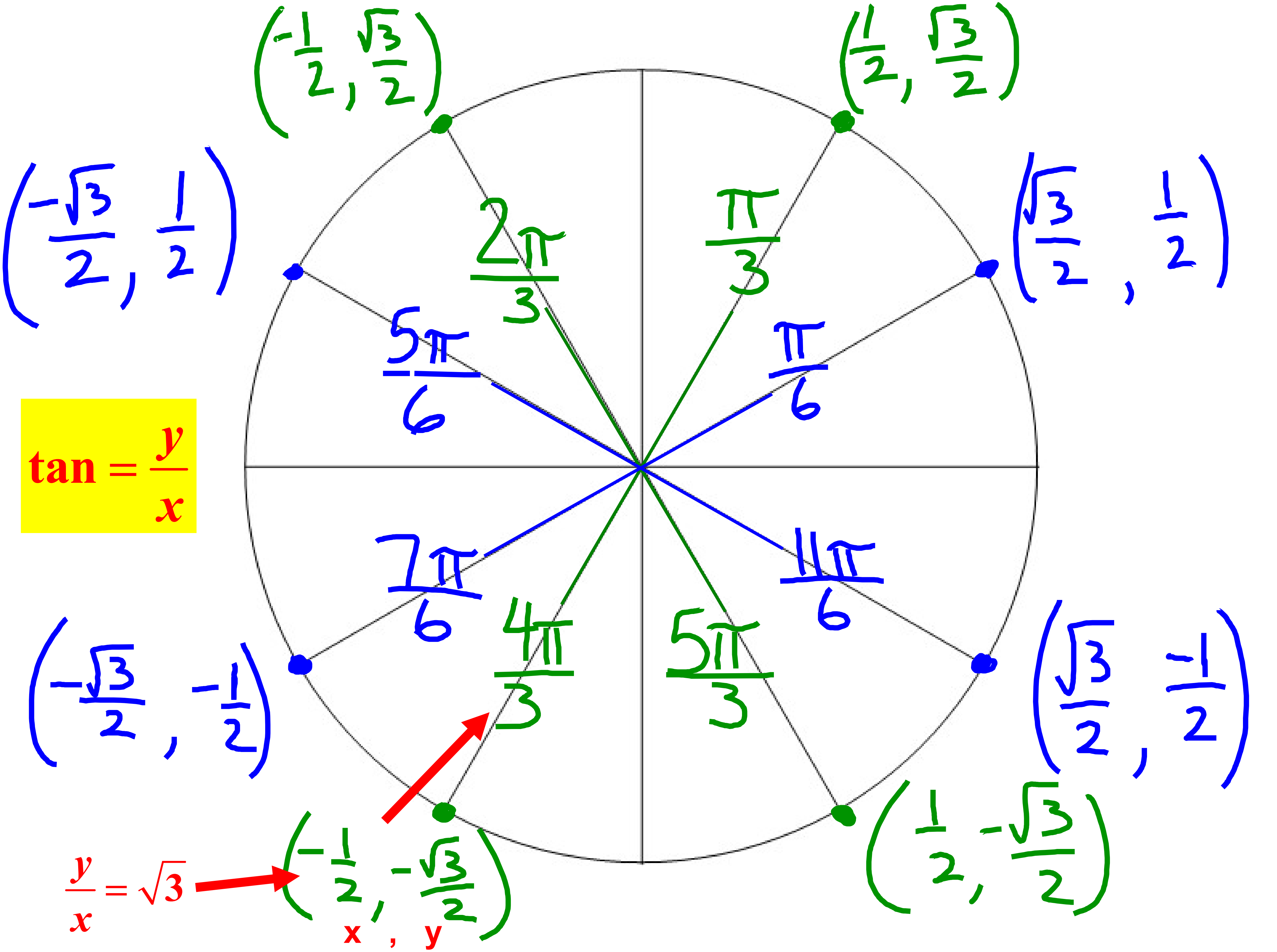
$$\tan\theta=\sqrt{3}$$

$$\theta=\frac{4\pi}{3}$$

↓ STOP HERE

Therefore, $-2-2\sqrt{3}\,i = 4\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

NOTE: if you simplify the expression on the right side, it will be equal to the value on the left side.



$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
$\tan = \frac{y}{x}$
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\frac{y}{x} = \sqrt{3}$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
x , y
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$